MATHEMATICS IN THE ETHICAL CULTURE HIGH SCHOOL.

By C. B. Walsh.

In giving a report of the mathematics in the Ethical Culture High School I do so with the fear that we of the school may seem to profess or at least to consider that we have discovered or attained the ideal. Let me preface the paper by saying that we are keenly conscious that there are defects both in the conception and in the execution of our course. Whatever I may say that sounds otherwise please pardon on the ground of enthusiasm. The purpose I hold in mind in giving this paper is an exchange of testimony—a practice which, to my way of thinking, is very helpful in all professions. Hence, I shall feel gratified if my paper simply provokes further discussion of the nature indicated. This exchange of testimony I consider a very encouraging sign of our generation. For example of what I mean look at the lists of problems found effective by various teachers now being published in *School Science and Mathematics.*

Moreover, in this paper I am going to deal with concrete details. It does not seem to me profitable to dwell on general principles before a group of professional teachers. You have long been familiar with such principles.

Let me group the reports under four main heads:

I. Aims of the Course.
II. Organization and Equipment of the Department.
III. Course of Study.
IV. Pupils' activities.

I. Aims.

The aims of our high school course are set forth in our pamphlet of the "Course of Study in Mathematics" better than I can state them. Hence, I shall merely quote from the same.

1. To furnish a solid foundation on which the more advanced work of college and technical school may be based.

2. To train pupils in habits of attention, accuracy and system, in dealing with mathematical material.

3. To develop an appreciation of the methods and spirit of pure science.

4. To give some conception of the importance of mathematical knowledge for understanding natural laws and for applying them to practical affairs.

Our methods of obtaining these aims, I hope, will be apparent from the details of the paper.

II. The Department—Organization and Equipment.

Although consisting of but few members our department is organized with the principal of the high school, Mr. Stark, at its head. It holds meetings about once a month. Twice a year, spring and fall, meetings of the department of the whole school, including elementary grades, are held. Topics of interest to all members are discussed at all meetings. For instance, suggestions for changes of text, reviews of new books, forms of written work, correlation within the department and with other departments, are among the topics discussed.

In accord with the spirit of the school from kindergarten through normal grades the organization is democratic. Any teacher is at liberty to bring up any topic which he or she believes would be of general interest and general profit.

The school provides a library for the department with an annual fund for additions. This library located in one of the mathematics recitation rooms is of free and easy access to the teachers and to pupils with the sanction of teachers.

The books include general reference and advanced books, histories of the subject, text-books, etc. *School Science and Mathematics* finds its place here, too. It is our hope to accumulate a very complete set of text-books used in foreign schools of secondary grade and a beginning of this collection has been made. To such texts pupils are occasionally referred when they are sufficiently conversant with other languages than English to make this a possible and profitable correlation of language work and mathematics.

Teachers find in this library books useful for the advanced study as well as material for daily service. A classified collection of problems, and sample test papers are also kept here.

Additions to this library are made at the requests of teachers.

The department has also started a mathematical museum.

This is still little more than embryonic, but is promising. It includes an old sextant picked up in a pawnshop on the Bowery and many crude instruments fashioned after the devices of our ancestors. These instruments the boys have made in the school shop. Among them may be found a baculus mensorius, pantagraph, astrolade, quadratus, and sector compasses.

It is the custom of our school to give an annual exhibit of its work, and recently at such exhibitions, while the entire work of the institution has been represented, each year one subject has been emphasized. Year before last the work of mathematics was especially exhibited. At that time, by means of exercises of pupils, printed reports, outlines, and summaries the work was displayed so as to show the continuous development from the kindergarten through the high school. The work was arranged both by grades and by topics. That is, besides showing the sequence by classes, groups of papers on graphic work, old instruments, real problems, etc., were shown. This material has been preserved under similar groupings.

A so-called current exhibit is always in place in the building by means of which specimen papers from each course are kept posted on large swinging leaves, and, as renewed, the older papers are put in portfolios so that the work of a course may be shown in continuous form to date at all times.

III. Course of Study.

The course of study covers four years and includes the usual algebra, plane and solid geometry, logarithms, plane and spherical trigonometry.

The first two years' work is required of all pupils graduating from the school. During these years substantial beginnings are made in algebra and geometry. Usually work in algebra to and including quadratics and in geometry to Book IV is accomplished.

The rest of the course is elective. Time is taken at all stages of the work and particularly during the first two years, *i. e.*, the prescribed work, to emphasize the history, general concepts and applications of the subject that those who end their mathematical training before the conclusion of the four years' course may carry away fundamental principles and facts that make for general culture. The elective work is divided when possible

into college and non-college classes. In the former stress is laid on theoretical or, if you will, pure mathematics—the regular college preparatory work. In the non-college classes more attention is given to applications. Constructive work and numerical problems receive special emphasis.

That students who conclude their course before its completion (*i. e.*, at the end of the second or third year) may have as broad an outlook on the subject as possible, mathematics is taught as such—not as arithmetic, algebra, geometry, etc. The so-called parallel method is pursued in teaching the branches of the great science. Emphasis at different periods is, of course, shifted. For example, the first year of high school, algebra receives most attention and during the second year geometry is in the foreground.

We place text-books in the hands of our pupils for two reasons: (1) as reference books and (2) as collections of problems. The method of class work is practically that used under the syllabus method. Pupils keep note-books for addenda. All written work passed in by the pupils is corrected, returned and preserved by the pupil, who is thus furnished at the end of his course with a sort of loose-leaf ledger note-book of the course.

A topic which seems to have received little general attention we include, *i. e.*, the history of mathematics. We do not give a special course in the history of the subject, but in a seemingly desultory way we develop the history along with the subject, often more as a point of view than as a topic. If, for example, a theorem in geometry is developed which has a significant history, that matter is called up.

We introduce the systematic study of demonstrational geometry by a rapid survey of the subject as an art, *i. e.*, tracing the empirical geometry of the Babylonians and Egyptians in its evolution to the geometry as a science among the Greeks. We believe that by pointing out the crude ideas of our remote ancestors, by following the evolution of new facts, by naming the great masters of our science and their discoveries, we can teach our pupils what general procedure to avoid and what to follow, and emphasize the fact that there is no "royal road" to mathematics; that its development is the result of much

labor; that the science is growing and hence is alive. Such facts tend to enlist the sympathy of the students with the true spirit of science.

As a device for fostering the individuality of pupils, we not infrequently have them develop and present to the class original theorems and proofs. We require pupils at times to look up special topics and report—as, for example, the various historical proof of the Pythagorean proposition. Occasionally for this work it is necessary for the pupil to use a French or a German book. This is a helpful correlation of subjects.

Frequent summaries and outlines are required. The ability to collect and systematize information is a scientific instinct which it is well to foster. Perhaps it may be a question of summarizing a month's or a semester's work. Perhaps it may be important to classify equations and epitomize the method of solution.

A feature of our work which we believe very practical is the field and general work with instruments. After all one of the chief functions of mathematics is measurements—be it of land or of time. In the grade school measurements of distances with the steel tape in and out of the building is begun. In the high school, as time permits, field work in measurements of distances and areas is practiced. The school possesses an excellent transit which is used by the older pupils for such purposes, but before that stage is reached the tape and rod are used. Besides, use is made of the crude instruments, now obsolete or at least obsolescent. I have referred to these instruments earlier in the paper and, hence, shall not dwell on them here—except to say that they prove especially effective for this work for the reason that, devoid of the elaborate mechanism found in modern instruments for accurate adjustments, they illustrate clearly the application of geometrical principles.

Throughout the high school course we aim to make prominent the applications of mathematics to practical affairs. Our chief device in this direction is the real problem. We have a collection similar to that published by Horace Mann School in *Teachers College Record* for March, 1909.

In the early part of the algebra formulas from geometry, physics, engineering, etc., form the basis of this work.

Permit a few illustrations.

Given the fact that the sag and length of trolley-wires are related by the formulas, $S = wl^2/8t$ and $l' = l + 8S^2/3l$ where S = sag in feet, w = weight of wire in lbs. per ft., l = distance in feet between supports, t = tension of wire in lbs., and l' = actual length of wire between supports in feet.

Pupils are required to transform these formulas in various ways, to combine them, and to evaluate them under different sets of data and to tabulate the results of evaluation.

Again, the students are told that a sinking fund is a sum of money set aside annually at compound interest to liquidate a debt. C = number of dollars debt, r = rate of interest, S = sum set aside, n = number of years, we have the relation,

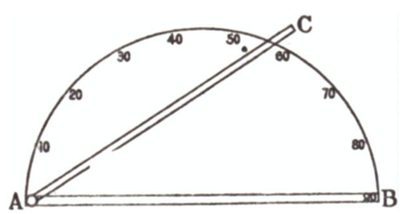
$$C = \frac{S[(1 + r)^n - 1]}{r}.$$

From this determine what sum must be set aside annually as a sinking fund by a government owing $500,000 to liquidate its debt at the end of four years, money yielding 5 per cent.

Again—the time of one vibration of a pendulum is given by the formula $t = 16\sqrt{l}$, where t is measured in seconds, and l, the length of the pendulum, is measured in inches. Make a graph showing the relation between the length of a pendulum and its time of vibration. From the graph determine the length of a pendulum which will beat seconds and the time of vibration of a pendulum 3 feet long. Check by calculation.

The work with the instruments described gives rise to various problems, sometimes merely illustrations of geometrical facts, for example:

The following is a diagram of an instrument used for measuring the altitude of the sun or of a star. AB is held hori-



zontally and the pivoted rule AC is aimed at the sun or star. Show that the semicircular scale must be divided into 90 equal parts in order that the edge of the rule may indicate the required number of degrees of altitude.

Again—to measure the height of a church spire, a rod 10 feet long is planted vertically at position *A*, then at position *B*. The observer takes such a position that the top of the spire, the top of the rod, and his eye, are all in line when he stands erect. He measures the distance from this position to the rod, doing this for each of the two positions of the rod, obtaining the measurements 4 feet and 8.3 feet. He also measures the distance between his two positions, finding it to be 138 feet. If the observer is 6 feet tall what is the height of the spire? Explain your solution.

We have groups of problems of this nature for various topics and also lists which are frankly miscellaneous. Again, to be concrete, I quote from a list on similar triangles.

To determine the distance of an island off shore, a stake is set on shore at the nearest point to the island. A line is laid off on the shore at right angles to the line from the stake to the island. At any convenient point *C* in the second line, a third line is laid off at right angles to the line from *C* to the island. Then a stake *D* is set in line with the first and third lines. What distances must be measured? Assume distances and calculate results.

We also have a few problems that lend themselves more readily to graphic solution than to any other means.

Take one illustration. About the time that the Pacific Railroad was opened the newspapers passed around the following question: Suppose that it take a train just one week to run the whole length of the road, and that one train leaves each end of the road each morning. How many trains will a person meet in going the length of the road, not counting the train which arrives as he starts nor the one that starts as he arrives?

Before concluding this part of the paper I want just to touch on two topics which doubtless occur to many here: viz., (1) our method of dealing with "limits" and (2) our use of the graph. On both these points we seek to take a natural, middle course. With regard to limits perhaps a cursory suggestion will be sufficient to indicate our procedure. We deal with the topic on the basis of successive approximations. That is to say, in dealing with the "incommensurable cases" we show that by continuing to change our unit of measurement for one

smaller we can get a negligible remainder and, hence, for all practical purposes, *i. e.*, to any degree of accuracy desired or demanded we can establish the proposition.

The graph we use whenever its use serves to make the problem in question clear or when, as in the illustration already cited, the graphic method of solution is the simplest. We avoid, however, bringing in the topic for its own sake. We use graphs, then, as illustrations, when it is actually desirable in a problem, or when approaching an old problem from a new point of view such as the graph offers, helps to enforce the idea.

Again—before leaving considerations regarding the course of study, I might append a note concerning our grading system. We have half-year classes, so that a pupil failing or entering in the middle of the year need not go back a whole year in the work but has the advantage of a flexible system. Frequently, too, we have two divisions of the same class, into a brighter and a duller section—although, of course, not so labelled.

We have, then considered the aims of our department, its organization and equipment and the course of study. Let us now glance at the students' activities not directly in the course.

IV. Students' Activities.

We have a mathematics club. This is a student organization meeting fortnightly for the consideration of topics which might be classified as "mathematical recreations" and various topics not possible or desirable to introduce into the regular course. The organization is small but interested. Our subject is not one to attract the masses, but is one whose followers have always been devoted. A plan is on foot to increase the membership in this organization by inviting members of other schools in the vicinity interested in the subjects to join.

I can best give you an idea of this organization by naming some of the topics discussed at its meetings: (*a*) History of Notation in Some of its Interesting Phases, (*b*) Short Cuts in Arithmetic, (*c*) Algebraic Fallacies, (*d*) Geometric Fallacies, (*e*) Famous Geometry Problems, (*f*) The Slide Rule, (*g*) Magic Squares, (*h*) Sector Compasses, (*i*) Sextant.

Other topics proposed are: (*k*) Interesting Facts from the Theory of Numbers, (*l*) Shadow Geometry, (*m*) Sun Dials.

Some of these topics occupy more than one meeting. Often

more than one pupil is scheduled to take part and all are urged and usually do take part in an active discussion of the topic in hand.

Our high school has an assembly two or three times a week. A committee of teachers and pupils arrange programs for these assemblies. It is the intention to have one of these assemblies each week conducted by a member or members of the school, teacher or pupils. Some such assemblies have been given in charge of a mathematics class or club.

Last year, for instance, the mathematics club gave an assembly on "Mathematics used in the Various Professions." This year, Columbus Day, the senior mathematics class gave an assembly in which they discussed the instruments used in navigation in the time of Columbus. These assemblies have proved interesting and instructive both to participants and auditors.

You know how dangerous it is to ask a person who has been abroad to tell you about the trip. I fear I have done much to show that it is just as dangerous to ask a person to tell something about his work. Hence, to avoid further trespassing on your patience I shall omit other details.

In brief, then, our high school course in mathematics with its fourfold aim of preparation for advanced work, training for accuracy, developing of scientific appreciation, and application to practical affairs, is in the hands of an organization department with an equipment which includes a library and incipient museum. Our course of study, two years' required and two years' elective work, is one in mathematics, not in algebra, geometry, etc., as distinct branches, thus keeping the broad point of view ever in the foreground. As devices to further our aims we require frequent summaries; we include field work with modern and ancient intruments; we lay stress on the real problems. We seek to stimulate the students' interests by encouraging a mathematics club and giving into the hands of pupils in the department some of our high school assemblies.

I am conscious of having included much that is commonplace. Perhaps to some of you, our system, as I have unfolded it, contains nothing new. I hope to many there will be at least one new idea to carry away.

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